## Improving the Visual Magnitudes of the Planets in *The Astronomical Almanac*. I. Mercury and Venus

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### ABSTRACT

Estimates for the apparent V magnitudes of the planets currently published in  $The\ Astronomical\ Almanac$  are based on phase coefficients,  $\Delta m(i)$ , presented in Harris (1961) along with values for V(1,0) from de Vaucouleurs (1970). Work is currently underway to update these values. The apparent V magnitudes of Mercury and Venus are examined here. This analysis provides new values for V(1,0) and  $\Delta m(i)$  derived from a variety of V photometric data sets for both Mercury and Venus. New data show that the previous value of V(1,0) for Venus was approximately 0.10 mag too faint because the small aperture used with photoelectric tubes not did not capture all of the light from Venus' relatively large disk. The Venus photometry also shows an abrupt and distinct "tail" beginning at a phase angle of about  $160^{\circ}$ , that is the curve abruptly changes direction somewhere between a phase angle of  $160^{\circ}$  and  $165^{\circ}$  and begins ascending. Circumstantial evidence suggests that this tail is caused by sunlight forward scattered through Venus' atmosphere. The RMS scatter in the calculated magnitudes was found to be 0.10 mag for Mercury and 0.07 mag for Venus.

Subject headings: Planets and satellites: individual (Mercury, Venus)

### 1. Introduction

The calculated apparent V magnitude of a solar system body such as a planet is given by the equation (Harris 1961)

$$V = V(1,0) + 5\log(rd) + \Delta m(i) \tag{1}$$

where V(1,0) is the magnitude of the body as seen at a distance of 1 AU from both the Sun and the Earth and a phase angle of  $0^{\circ}$ , r is heliocentric distance of the planet in AU, d is its geocentric distance in AU, and  $\Delta m(i)$  is the correction for the phase angle, i.

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Form Approved OMB No. 0704-0188 The phase angle is the angle between the observer and the Sun as seen from the center of mass of the body. This determines the portion of the body seen by the observer lit by the reflected light of the Sun. For an observer on the Earth, the phase angle is usually calculated for an observer situated at the geocenter. Except for very near objects, such as the Moon, the difference in the phase angle is insignificant for a theoretical observer at the geocenter and an actual observer on the surface (Hilton 1992).

The quantity  $\Delta m(i)$ , the phase coefficient, is measured empirically. In practice, the phase coefficient is modeled by a polynomial relation whose coefficients are determined from observations of the planet. Except for the inferior planets and Mars (Young 1974), a linear relation is sufficient to determine the phase coefficient.

Since at least the 1984 edition of *The Astronomical Almanac*, the phase coefficients used for the planets are based on those presented in Harris (1961). Harris did not determine the coefficients but was reporting the work of others. The phase coefficients of Mercury and Venus, for example, were determined by Danjon (1949) with a correction for Mercury in Danjon (1950). The values for V(1,0) are from de Vaucouleurs (1970). These V(1,0) values are similar to those presented in Harris for some of the planets, but others differ significantly. The de Vaucouleurs values of V(1,0) for Mercury and Venus are -0.42 mag and -4.4 mag, respectively.

A preliminary re-evaluation of the Danjon (1949) observations of Mercury and Venus using the de Vaucouleurs (1970) correction to the V magnitude showed that it was statistically difficult to support the use of cubic relations to describe V(1,0) and phase coefficients for these planets. Beginning with the edition for 2005, The Astronomical Almanac began using quadratic relations for Mercury and Venus. However, the re-evaluation of the data also found that the new relation for the phase coefficient for Mercury produces values that are  $0.13 \pm 0.07$  mag brighter than the previous one and the relation for Venus produces values that are  $0.12 \pm 0.07$  mag fainter. The largest differences are at the greatest phase angles.

Thus, it became necessary to ask whether the planetary magnitude parameters for Mercury and Venus used in *The Astronomical Almanac* need to be updated in light of more recent photoelectric and CCD V band observations. Section 2 discusses the values of V(1,0) and  $\Delta m(i)$  for Mercury; Section 3 discusses those values for Venus; and Section 4 presents the conclusions.

### 2. Mercury

The phase function for Mercury from Danjon (1949), corrected by Danjon (1950) with V(1,0) determined by de Vaucouleurs (1970) is

$$V(1,0) + \Delta m(i) = -0.42 + 3.80 \frac{i}{100} - 2.73 \left(\frac{i}{100}\right)^2 + 2.00 \left(\frac{i}{100}\right)^3.$$
 (2)

Danjon determined the photovisual phase function from 225 observations of Mercury made between Oct. 15, 1937 and May 22, 1948 covering a phase angle of  $3^{\circ} < i < 123^{\circ}$ . The difference between the V and photovisual magnitude was determined by de Vaucouleurs (1970) to be -0.17 magnitudes. This relation was used in *The Astronomical Almanac* prior to the edition for 2005.

More recently, Irvine et al. (1968a) made 31 V band observations of Mercury between June 15, 1963 and May 17, 1965 covering phase angles from 58° to 115°. And Mallama et al. (2002) made 24 observations using the SOHO/LASCO instrument (henceforward LASCO) between Sept. 9, 1999 and July 8, 2000, from 2°.1 to 5°.1 and from 160°.0 to 169°.5, and 84 ground-based observations between June 6, 1999 and Mar. 3, 2000, covering phase angles of 4°.7 < i < 137°.2. Figure 1 shows the Irvine et al. (1968a) and Mallama et al. (2002) observations of Mercury as a function of phase angle. Both data sets are in agreement.

The next step is to make the best determination of V(1,0) and  $\Delta m(i)$  from these data. This was done using a standard least-squares fit to the data. Tests for the appropriate order of the polynomial solution were made by first examining the covariance matrix for large cross-correlations and then using a  $\chi^2_{\nu}$  ( $\chi^2$  per degree of freedom) test. The  $\chi^2_{\nu}$  test is sensitive to both the discrepancy between the estimated and true function and the deviations between the data and the parent function. Thus, it can be ambiguous, so an  $F_{\chi}$  test of an additional term (Bevington & Robinson 1992) was also used to determine whether the addition of another order to the phase coefficient made a significant improvement to the fit. In all cases, the  $F_{\chi}$  test gave no additional information beyond confirming the  $\chi^2_{\nu}$  test.

Table 1 gives the best-fit set of coefficients for various data sets. None of the authors provided estimates for the uncertainty in the photometry, thus the best-fit RMS scatter of 0.10 mag was used in determining the value of  $\chi^2_{\nu}$ . Thus, the results of the  $\chi^2_{\nu}$  test is mainly an indication of the departure of the fit from the optimum as a function of the phase angle. However, the minimum value of  $\chi^2_{\nu}$ , 1.04, for the best fit to the weighted data is near 1, indicating that the estimate for the uncertainty in the data is approximately the estimated value of 0.10 mag These results lead to several observations:

• It is impossible to actually observe Mercury at a phase angle of 0° and a distance of

- 1 AU from both the Earth and Sun. Thus, the value of V(1,0) is simply the constant coefficient of the fitted polynomial and can vary significantly depending on the order of the fit.
- The fit using the Danjon (1949) is only quadratic, unlike the cubic fit determined by Danjon (1949, 1950). The reason for this reduced order is that the  $\chi^2_{\nu}$  and  $F_{\nu}$  tests indicated that a cubic fit was not justified. The coefficients for the Danjon data in Table 1 have been used in *The Astronomical Almanac* for the physical ephemeris of Mercury beginning with the edition for 2005.
- The Mallama et al. (2002) LASCO data coverage near a phase angle of 180° requires at least a third-order polynomial to provide a good fit. However, the LASCO data alone do not provide enough information for a unique cubic polynomial.
- All three data sets provide similar values for the RMS spread of the (O − C)s. Considering the Danjon (1949) data are 50 to 60 years older than the Mallama et al. (2002) data, this would indicate that the RMS spread in the (O − C)s is not caused by the uncertainty in the measurements, but rather are a result of the natural variability in the apparent V magnitude of Mercury. The LASCO instrument is spacecraft based, thus this variation can not be the result of atmospheric perturbations. Instead the variation must be the result of albedo variations on Mercury itself. The hermian sub-Earth longitude was not included in the model. The 0.10 RMS spread is just slightly larger than the extremes of the longitude binned RMS spread of Mallama et al. (2002). At least some of the smaller RMS spread by Mallama et al. is the result of using a seventh order polynomial which could not be justified statistically. The rather large RMS 0.16 mag spread of the LASCO data alone in table 1 is an artifact of the low order fit required by the statistical tests. Thus, the RMS spread in the observations can be explained by albedo markings on Mercury's surface.
- Changing the order of the polynomial does significantly change the values of the coefficients, that is, the higher-order (>3) fits have significant non-zero cross-correlations.
- Over the phase interval  $3^{\circ} < i < 123^{\circ}$  the mean V between the polynomial determined from the Mallama et al. and Irvine et al. observations and the polynomial determined by Danjon (1949, 1950) (see Fig. 1). However, over the interval  $123^{\circ} < i < 169^{\circ}.5$ , which is outside the valid range of the Danjon (1949) observations, the mean V was 0.20 mag fainter.

Based on these least squares fits it was decided that the most representative polynomial

for  $V(1,0) + \Delta m(i)$  for Mercury is:

$$V(1,0) + \Delta m(i) = -0.60 + 4.98 \frac{i}{100} - 4.88 \left(\frac{i}{100}\right)^2 + 3.02 \left(\frac{i}{100}\right)^3.$$
 (3)

### 3. Venus

The  $\Delta m(i)$  of Venus currently used in *The Astronomical Almanac* was determined by Danjon (1949) and the V(1,0) was determined by de Vaucouleurs (1970). These values were determined from 335 observations of Venus between Oct. 3, 1937 and Sept. 15, 1947. The expression used in *The Astronomical Almanac* is

$$V(1,0) + \Delta m(i) = -4.40 + 0.09 \frac{i}{100} + 2.39 \left(\frac{i}{100}\right)^2 - 0.65 \left(\frac{i}{100}\right)^3.$$
 (4)

where the range of phase observed by Danjon was 0.9 < i < 170.7. Danjon's data in the photovisual system and the fit curve, corrected to V are shown in Fig. 2. This relation was used in *The Astronomical Almanac* prior to the edition for 2005.

Since Danjon, there have been four other studies of Venus' magnitude as a function of phase: Knuckles et al. (1961), Irvine et al. (1968a), Irvine et al. (1968b), and Mallama et al. (2004, private comm.).

Knuckles et al. (1961) made 56 observations of Venus between June 4, 1954 and Oct. 20, 1960 covering phase angles from 15°.9 to 171°.5. They estimated the phase function drawing by eye what they determined to be the best-fit line through the observations and tabulating the result. Their tabulation gives values for the phase function similar to those from Danjon's algorithm. However, the V(1,0) determined from Knuckles et al. is 0.10 mag brighter than Danjon.

Irvine et al. (1968a) made 78 observations from May 2, 1963 through Dec. 8, 1965 covering phase angles from 36°0 through 157°3. Irvine et al. (1968b) made six observations between Aug. 24, 1964 and Aug. 11, 1965, covering phase angles between 41°3 and 93°2. Since this second data set is so small, it was only used in combination with the Irvine et al. (1968a) data.

Mallama (2004) made 222 observations between May 10, 1999 and June 12, 2004 covering phase angles from 2°.2 to 170°.2. These observations are available from the observer on request.

The best least-squares polynomial fits for each of the data sets are given in Table 2.

### As with Mercury:

- None of the authors provided estimates of the uncertainty in the observations. Thus, the RMS scatter, 0.07 mag, of the best-fit model for Venus was chosen as the estimate for the uncertainty in the data for use in the  $\chi^2_{\nu}$  test. The value of 0.98 for  $\chi^2_{\nu}$  of the best fit model indicates that the estimated value for the uncertainty is approximately correct.
- The  $\chi^2_{\nu}$  and  $F_{\nu}$  tests of the Danjon (1949) data did not support the use of a cubic relation for  $\Delta m(i)$  for Venus. Instead a quadratic relation was adopted. The coefficients in Table 2 are the ones that have been used in *The Astronomical Almanac* since the edition for 2005.

Fig. 3 shows the Knuckles et al. (1961) data with its best-fit polynomial. The RMS scatter in the observations is nearly twice that of the other two data sets. The scatter is particularly large for phase angles less than 30°, so the best value of  $\chi^2_{\nu}$  for this data set is 6.10. Based of the relatively poor quality of the fit, the Knuckles et al. data was not used for the final determination of V(1,0) and  $\Delta m(i)$  of Venus.

Comparison of the different fits to the Irvine et al. data, Fig. 4, with the Mallama data shows a difference in V of about 0.1 mag The difference is due to the small apertures used with the photoelectric tubes, which did not capture all of the brightness of the comparatively large disk of Venus (Mallama 2004, private communication). Trial solutions using both the Irvine et al. and Mallama data produced solutions with significantly greater RMS scatter, 0.10 mag, than the RMS scatter, 0.07 mag, of either of the individual data sets. Thus, the Irvine et al. data was rejected.

Considering that the Danjon data is 50 to 60 years old and was collected using the photovisual system, which requires an uncertain correction, the final, best fit solution for V(1,0) and  $\Delta m(i)$  for Venus is determined solely from the Mallama data. This data is shown in Fig. 5.

The first thing to note is that there is a perceptible increase in the RMS scatter of the observations with phase angles less than about 20°. At small phase angles Venus is at its greatest distance from the Earth and near the Sun. Thus, observing it is difficult and it is not surprising that the RMS scatter should increase. This increase in the RMS scatter can also be seen in the Danjon (1949) (Fig. 2) as well as the Knuckles et al. (1961) (Fig. 3) data. Dropping the data at small phase angles did not significantly improve the fit ( $\chi^2_{\nu} = 0.98$  with the data, = 0.96 without it), nor significantly change the values of the relation. Thus, the final fit includes the data at small phase angles.

The second phenomenon to note is, except for the two observations at a phase angle of 167°, the curve abruptly changes direction somewhere between a phase angle of 160° and 165° and begins ascending! As with the observations at  $i < 20^{\circ}$ , the observations at  $i > 160^{\circ}$  must be made with Venus near the Sun. Thus, there are difficulties in making such observations. However, the change in the relation is quite distinct and does not have the character of simply being an increase in the RMS scatter as seen at small phase angles.

The likely cause of the change in the phase curve is the forward scattering of sunlight by Venus' atmosphere. The evidence for this conclusion is circumstantial, but compelling:

- This phenomenon is *not* seen in the V curve for the atmosphereless Mercury.
- The Danjon (1949) data also appears to show something of a change in direction at high phase angles. The change in direction is not as prominent in the Danjon data and could be interpreted an increase in the RMS scatter caused by the difficulties in making observations near the Sun. However: 1. The direction of the change in the data is consistent with the tail of the phase curve seen in the Mallama data (i.e. the "scatter" is towards a brighter value for the phase coefficient and results in a high negative value for the cubic term in Danjon's relation. 2. A large portion of the increase in the light coming from forward scattered sunlight consists of a faint light distributed along the rather long "unlit" limb of Venus and would be faint in comparison to the lit crescent.
- Both the Danjon and the Mallama datasets contain observations that follow the trend of the data at smaller phase angles. This sort of behavior would be expected if the faint forward scattered light from the "unlit" edge of Venus became difficult to observe due to weather conditions at the observation site. In the case of the Mallama data, the two observations (at a phase angle of 167°) which follow the trend of the data at smaller phase angles were both made on the same day and were the only observations made on that day. Thus the failure to observe the forward scattered sunlight in the Mallama data is consistent with the supposition that it is the result of weather conditions.
- The Knuckles et al. (1961) data might show a "tail" as well. However, this data set contains only three observations with  $i > 160^{\circ}$ . Thus, considering the large RMS scatter at small angles, this evidence is problematic.

Because of the rather small number of observations, 15, made between 159° and 166° and the RMS scatter of the data it was not possible to determine the point at which the phase curve began to change more precisely than the interval between  $160^{\circ} < i < 165^{\circ}$ . Nor was it possible to fit a spline curve to the data. Thus, the main part of the phase curve, given in Table 2 was fit using the 209 observations in the interval  $2^{\circ}2 < i < 164^{\circ}7$ . The tail

of the phase curve was fit using the 21 observations in the interval 160°.1 < i < 170°.2 left after dropping the two discrepant observations at i = 167°. The tail segment of the curve had few enough observations and was short enough that only a linear relation would fit it statistically. The final relation for Venus is:

$$V(1,0) + \Delta m(i) = \begin{cases} -4.47 + 1.03 \frac{i}{100} + 0.57 \left(\frac{i}{100}\right)^2 + 0.13 \left(\frac{i}{100}\right)^3, & 2^{\circ}2 < i < 163^{\circ}6 \\ 0.98 - 1.02 \frac{i}{100}, & 163^{\circ}6 < i < 170^{\circ}2 \end{cases}$$
(5)

The choice of 163.6 as the junction between the two segments of the phase curve is arbitrary. It is the phase angle at which the two curves crossed. The RMS scatter in the V magnitude is 0.07 mag for the main part of the phase curve and 0.05 mag for the tail. The main part of the new relation compared with the Danjon using the de Vaucouleurs (1970) correction from photovisual to V is 0.10 mag brighter.

### 4. Conclusions

The Mallama et al. (2002) and Irvine et al. (1968a) observations of Mercury and the Mallama (2004) observations of Venus provide new relations for the apparent V magnitude of these two planets as a function of phase. The new relations, based entirely on V band observations, also provide an estimate of the RMS scatter in the calculated visual magnitudes of these two planets, 0.10 mag for Mercury and 0.07 mag for Venus.

The Knuckles et al. (1961) observations of Venus were not used in the final determination of the apparent V magnitude of Venus because these observations were determined to have a large RMS scatter in their low phase angle observations. Similarly, the Irvine et al. (1968a,b) observations were not used because the use of a small aperture with photoelectric tubes failed to capture all the light coming from the relatively large disk of Venus. The new relations produce calculated visual magnitudes that differ significantly from the quadratic relations based on the Danjon (1949) observations currently used in *The Astronomical Almanac*.

The valid range for the relation for Mercury is  $2^{\circ}.7 < i < 169^{\circ}.5$ . For  $3^{\circ} < i < 123^{\circ}$  (the valid range of the Danjon (1949, 1950) relation) the mean V of the new relation is 0.07 mag brighter. If the old relation is extended to 169°.5, the mean V of the new relation over  $123 \deg < i < 169^{\circ}.5$  is  $0.20 \deg$  fainter.

The phase curve for Venus consists of two segments. The valid range for the new main part of the relation for Venus is  $2^{\circ}.2 < i < 163^{\circ}.6$ . The second segment covering  $163^{\circ}.6 < i < 170^{\circ}.2$  was required because at high phase angles Venus begins to become brighter as a function of increasing phase. The suggested cause for this brightening is sunlight forward

scattered by Venus' atmosphere. The data set was small enough that the point at which the forward scattered sunlight began to become significant could only be determined to be in the interval  $160^{\circ} < i < 165^{\circ}$ , nor could the transition be fit with a spline curve. The end of the main phase curve segment and the beginning of the tail segment at 163°6 was arbitrarily chosen as the point where the solutions for the two segments crossed. The mean V of the new relation is 0.10 mag brighter than the old one.

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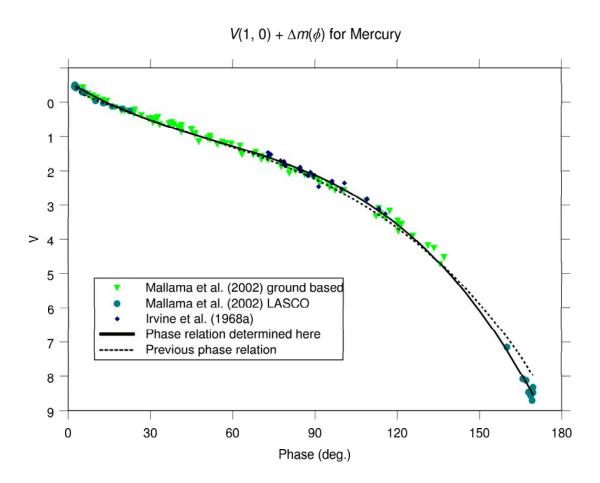


Fig. 1.— Apparent magnitude at unit distance of Mercury as a function of phase angle. The solid line is the best-fit value for  $V(1,0) + \Delta m(i)$  given in Eq. 3. The dashed line is the old relation using V(1,0) from de Vaucouleurs (1970) and  $\Delta m(i)$  Danjon (1949, 1950).

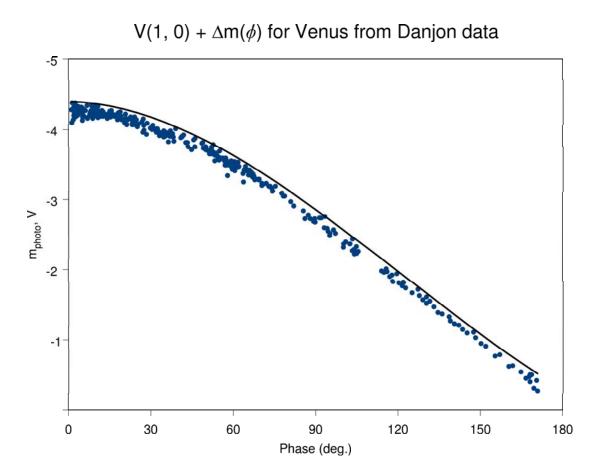


Fig. 2.— Apparent magnitude at unit distance as a function of phase angle from Danjon (1949) data. The solid line is the best-fit value for  $V(1,0) + \Delta m(i)$  given in eq. 4. The data is on the photovisual system. Thus, the difference between the data and the line is the correction from photovisual to V (de Vaucouleurs 1970).

### $V(1, 0) + \Delta m(\phi)$ for Venus from Knuckles et al. data

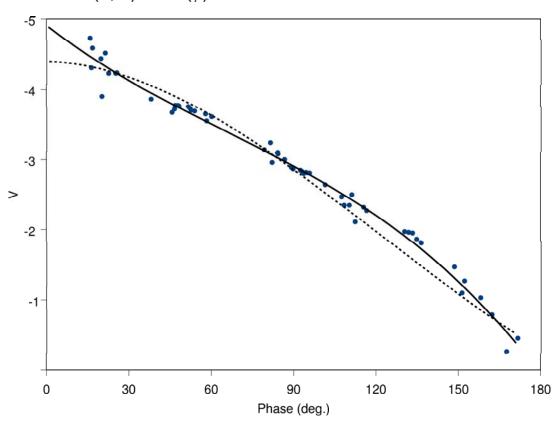


Fig. 3.— Apparent magnitude at unit distance of Venus as a function of phase angle from the Knuckles et al. (1961) data. The solid line is the best-fit value for  $V(1,0) + \Delta m(i)$  for this data set. For this data set the RMS scatter is 0.12 mag and the  $\chi^2_{\nu}$  is 6.10. The dashed line shows the old relation (eq. 4).

## A STATE OF THE PARTY OF THE PAR

 $V(1, 0) + \Delta m(\phi)$  for Venus from Irvine et al. data

-5

-4

-3

-2

-1

0

30

Fig. 4.— Apparent magnitude at unit distance of Venus as a function of phase angle from the Irvine et al. (1968a) (circles) and Irvine et al. (1968b) (squares) data. The line is the best-fit value for  $V(1,0) + \Delta m(i)$ . The dashed line shows the old relation (eq. 4).

90

Phase angle (degrees)

120

150

180

60

# $V(1, 0) + \Delta m(\phi)$ for Venus from Mallama data

Fig. 5.— Apparent magnitude at unit distance of Venus as a function of phase angle from the Mallama (2004) data. The solid line is the best-fit value for  $V(1,0) + \Delta m(i)$  given in eq. 5. The dashed line shows the old relation (eq. 4) fit to the Danjon (1949) data.

Phase (deg.)

Table 1. Best fit for V(1,0) and  $\Delta m(i)$  for Mercury using different data sets.

Data Set	Order of Fit	V(1,0)	Order of Coefficient of $\Delta m(i)$			RMS(V)
			$\frac{1}{1}$	2	3	
		mag	mag deg. <sup>-1</sup>	mag deg. <sup>-2</sup>	mag deg. $^{-3}$	mag
Danjon (1949)	2	-0.37	0.0212	$0.81 \times 10^{-4}$		0.12
Irvine et al. (1968a)	1	-0.24	0.0381			0.08
Mallama et al. (2002)	3	-0.60	0.0494	$-4.77 \times 10^{-4}$	$2.97 \times 10^{-6}$	0.11
Mallama et al. (2002) <sup>a</sup>	2	-0.47	0.0282	$1.44 \times 10^{-4}$		0.16
Mallama et al. & Irvine et al.	3	-0.60	0.0498	$-4.88 \times 10^{-4}$	$3.02 \times 10^{-6}$	0.10

<sup>&</sup>lt;sup>a</sup> LASCO data only.

Table 2. Best fit for V(1,0) and  $\Delta m(i)$  for Venus using different data sets.

Data Set	Order of Fit	V(1, 0)	Order of Coefficient of $\Delta m(i)$			RMS(V)
			1	2	3	
		mag	mag deg. $^{-1}$	$^{\mathrm{mag}}$ deg. $^{-2}$	mag deg. $^{-3}$	mag
Danjon (1949)	2	-4.35	0.0097	$0.86 \times 10^{-4}$		0.05
Knuckles et al. (1961)	3	-4.91	0.0303	$-1.64 \times 10^{-4}$	$0.83 \times 10^{-6}$	0.12
Irvine et al. (1968a)	2	-4.11	0.0014	$1.32 \times 10^{-4}$		0.07
Mallama (2004) <sup>a</sup>	3	-4.47	0.0103	$0.57 \times 10^{-4}$	$0.13 \times 10^{-6}$	0.07
Mallama (2004) <sup>b</sup>	1	0.98	-0.0102			0.05

<sup>&</sup>lt;sup>a</sup>2°.2 < i < 163°.6

 $<sup>^{\</sup>rm b}163^{\circ}\!\!\!.\,6 < i < 170^{\circ}\!\!\!.\,2$